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THE CELEBRATED

THEORY OF PARALLELS.

DEMONSTRATION

OF

THE CELEBRATED THEOREM.

EUCLID I, AXIOM 12.

With Appendix containing the philosophy of the demonstration, together with the partial refutation of Sir Wm. Hamilton's philosophy of the Unconditioned or Infinite.

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" But you forget that Geometrical Equality can do great things, both among gods and men."

—Plato. Gorgias.

Thus, also, universally, from the comparison of the equality of finites may be evolved some positive knowledge of the corresponding homogeneous Infinites, whether in Deity, space, time or degree.

—Appendix, Note A, Par. c.

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SKETCH

OF THE

ROMANTIC HISTORY OF PARALLELS.

(a.) Of all contested geometrical subjects that of *Parallels*—the great blemish in the immortal "*Elements*" of Euclid—is the most ancient as well as the most fascinating. At the porch of science and clasping infinite SPACE, the subject is at once simple and sublime. Its recorded history commences with the mild and the benevolent Euclid, 280 years B. C. (see Euc. I, 28;) and though forming an essential element in most demonstrations, yet, as an unestablished theory, it must have been handed down by the illustrious Pythagoras, by the venerable Thales, and probably by the Egyptian priests who were the instructors of these philosophers.

(b.) Says Lardner, "The *Theory of Parallels* has always been considered as the reproach of Geometry.* * *" In pamphlet published 1856, T. P. Thompson † declares, " * * * Ptolomey, Proclus, the Arab editor of Euclid, Clavius, Wolfius, Boscovich, D'Alembert, Thomas Simpson, Bonnycastle, Robert Simson, Varignon, Bezout, Leslie, Ludlam, Playfair, Franceschini, Legendre, La Grange, Le Croix, Bertrand, with others of later date have successively set their seals to the admission that till something was done on this point‡ Geometry was not entitled to the name of an exact science."

(c.) Stevelly, (Lond. Edin. and Dub. Phil. Mag. 1856, Vol. XII, p 220,) frankly acknowledges thus: "I was again induced to waste some hours on a subject on which in my schoolboy and college days I, in common I suppose with every schoolboy and collegian since the days of Euclid, had over and over again wearied myself in vain."

(d.) During the past half century the matter had been subjected to the most subtle analysis; the doctrine of *functions*, and the inadmissible doctrine of *limits* had been appealed to, but in vain. In the investigation of the subject, indeed, "many have lost much time, and some even their reason. * * *"§ Its investigations have embittered the most exalted friendships. In this research the philosophic minds of Leslie and Legendre have waged war. Finally many moderns concluded that "to deduce the property of two lines postulated as parallels involves a *direct* dealing with the *positive idea of infinity*—a task utterly beyond the reach of our faculties," &c.‖ Similar, but more elaborate declarations will be found in an article on *parallels* by James Adamson, D. D., in the London, Edinburgh and Dublin Phil. Mag., 1853, Vol. V, page 407; also in the Penny Cyclopædia, Vol. XVII, art. *parallels*, concluding paragraph, page 238, as also in other works. Thus at length did this mysterious subject powerfully aid in the founding of a false and atheistical philosophy,¶ and bid fair to remain a matter of contention during all the ages of the future. But God has been pleased to hearken to the voice

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†General T. Perronet Thompson, of Eliot Vale, Blackheath, London.

‡Meaning parallels.

§Encyclopedia Americana.—Subject, *Parallels*.

‖Lond. Edin. and Dub. Phil. Mag. 1857, Vol. XIII. page 413, as quoted from "Nichols' Cyclopædia of the Physical Sciences."

¶Thus, on the *hypothesis* that a *positive idea of the Infinite* is impossible to the human mind, false philosophers have contended that hence the existence of an Infinite Being is unsusceptible of demonstration, and is therefore a *superfluity*, and as such, an evil as an element of man's religious belief.

of the illustrious dead, and to reveal the *hiatus* of twenty ages as the reward of eight years of an Irish exile's diligent investigation. (See prop. B.)*

(e.) It is curious to observe that the history subsequent to the demonstration of prop. B. is no less romantic than that which precedes. Thus, in 1860, in order to bring this demonstration before the world, the author visited France, but in vain, owing to the mathematician Bertrand's absurd (and, as the author conceived, proud) demand of the custody of the manuscript. In June, 1864, the American Institute of New York refused to pronounce upon the demonstration. In December, 1864, the demonstration was deposited with Prof. Joseph Henry, of the *Smithsonian Institute*, Washington, D. C., who unphilosophically shrank from declaring an opinion, though twice entreated by the writer so to do. Prof. Henry finally announced the loss of the manuscript in the conflagration of February, 1865. Prof. Hildgard, of the Coast Survey, and others, have also shrank from expressing opinions. But peace must be given to the Divine Science, and Philosophers must not be debarred from the delight of knowing a beautiful truth through the jealousy, the unphilosophical timidity, or the prejudice of Bertrand, of Henry, or others: and therefore the Author publishes, at his own expense, (and out of a modest salary,) his establishment of the *Theory of Parallels*, and freely distributes the same throughout every land.†

(f.) The Author tenders his love to the Mathematicians and Philosophers of every nation, and in so doing confesses his conception that in Appendix, Note B, is pointed out an interesting field of geometric and philosophic thought; thus giving an impetus to man's approach to that ULTIMATE OF ALL TRUTH—the contemplation of the unveiled glory of that INFINITE INTELLIGENCE—that CAUSE OF ALL—the ETERNAL UNCAUSED.

MATTHEW RYAN.

WAR DEPARTMENT, (Office of Accounts,)
Washington, D. C., December 1st, 1865.

*For full information see "*Penny Cyclopædia,*" Vol. XVII; "*Ency. Americana;*" "*Lond. Edin. and Dub. Phil. Mag.,*" chiefly 1856–7; *T. P. Thompson's Geo.*, 1834; *Prof. Jas. Thompson's Euc.*; also, *Camerers' Euc.*, Berlin, 1825, &c.

†This pamphlet is but a fragment of the manuscript work, "The Perfect Geometry," &c., which contains the theories of the straight line and plane; also an Appendix containing the refutation of certain sophisms and metaphysical objections brought against prop. A, some original props., some *positive* knowledge of the Deity, &c. Academies desiring to publish may apply for this manuscript.

DEF. A. (GEOMETRICAL COINCIDENCE.)

Coincidence is the term given to the property of capability of exact occupation of the same space possessed by homogeneous and finite magnitudes, if *superposed.*

Cor. Coincidence assumes five forms, of which four are *direct* and one *indirect.*

1—The *immediate form,* as in the case of identical magnitudes. (Euc. I, 2, 4, 5, 8, 26, &c.)

2—The *transposition form,* as in dissimilar figures. (Euc. I, 35, 36, 37, 38, 42, 43, 44, 45 and 47; Euc. II, 11 and 14; and Euc. VI, 31, &c.)

3—The *repeated immediate form,* as where the several parts of one magnitude coincide at once respectively with the several parts of another.

4—The *continuous repeated immediate form,* as in the superposition of the circumference on the straight line, &c. (See prop. A.)

5—The *indirect form,* or *form of comparison with equal auxiliaries,* or *form wherein the magnitude of each thing is determined by identical conditions,* as in the case of A A′, B B′, fig. prop. A,* &c.

Schol.—Since a theorem is established (as all logicians admit) when its converse involves an impossible consequence; therefore, the certainty of coincidence in any case is established by inferring an impossibility from the contrary assumption.

N. B.—This simple schol. is the bulwark of prop. A.

DEF. B. (GEOMETRICAL EQUALITY.)

The capability of coincidence is called *equality.*†

Cor. 1. *Equality* can alone be predicated of finite magnitudes. [See Def. A.]‡

Cor. 2. (a.) The ideas of *equality* and *coincidence* are involved in one idea, "for equality is nothing but the capability of coincidence;"§ and since the idea of *coincidence* involves the idea of simple *motion;* therefore also the conception of *equality* involves the conception of simple *motion.* The same

*Thus is coincidence subject to that grand law of *variety* which is observable throughout all the operations of nature, and all the departments of knowledge, save alone the immutable laws of Geometry.

† " * * * from whence it is sufficiently apparent that by *Equality* is understood nothing else but a *possible Congruity.*" Barrow's Lectures No. XI.

‡" The observation that equality (meaning of magnitude not of ratio) implies being finite, is as old as the Philebus of Plato. 'And next, all the circumstances that appeared to be incompatible with it; as first, having *equal* or *equality;* and after that, having a *double,* or anything else that implies relation of number to number or measure to measure; all these we set down as appropriate to the finite only.'—Plato, *Philebus.* Plato's distinction will be found exact if limited to what he manifestly had in view. No man can attach any rational idea to *half* eternity; nor, by analogy, to *twice* eternity. If there are to be two co-existing eternities, (as for instance two rectangular parallelograms on contiguous bases, whose altitude is of unlimited length,) this is a question of *ratio;* which is quite another thing." T. P. Thompson's Geo. 5th edit. p. 132. Again, at page 153, "All references to the equality of magnitudes of infinite surfaces, in respect to the parts where they are avowedly without boundaries, are intrisically parallogisms; for it is tantamount to saying that boundaries coincide where boundaries are none." [This last is an immediate refutation of M. Bertrand's attempt to establish the properties of *parallels.* Author.]

§T. P. Thompson's Geo., 5th edit., 1834, page 148.

may be said of *inequality*, viz.: that its conception involves the conception of coincidence; that is, of partial coincidence, and therefore of simple motion, for *inequality* is simply the *equality of a part.*

(b.) *Motion* is inseparable from the drawing of a line, and its conception is inseparable from the conception of such. The conception of parallels (lines infinitely prolonged) creates the conception of motion.

N. B. —Axiom I involves the conception of motion.

ON THE TWO GEOMETRICAL AXIOMS.

Elucidation.—So beautifully has the ALL-WISE CREATOR framed the human mind, that whilst on the one hand, no *demonstrable* proposition can be intuitively perceived;* on the other hand, the power of intuitive perception is given respecting the truth of the *indemonstrable* propositions. Human reasoning being the deduction of theorems from the comparison of others, therefore, there must, at least, be two theorems, the evidence of the truth whereof is intuitive. Such are called *Axioms.*

Axiom I.—Any two finite and homogeneous magnitudes must be either equal or unequal.

Axiom II.—No magnitude can possess inconsistent properties.

Schol. From these two theorems, which the mind receives directly from the Deity, are evolved all the laws of Geometry.

THE THEORY OR PROPERTY OF PARALLELS.

PROPOSITION A. THEOREM.

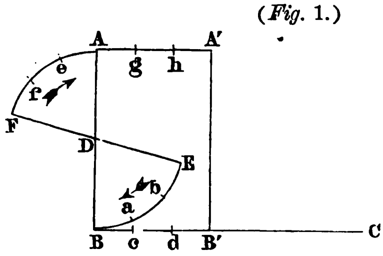
[See Note A, Appendix, for the philosophy of this demonstration.]

If a straight line (A B) *which intersects another* (B C) *move on that other in the same plane for a given length* (B B′), *and so* (1) *that it shall always retain a given inclination to that other, and* (2) *that the same point* (B) *in it shall always lie in that other; then shall the locus* (A A′) *of any point* (A) *in the moving line be equal to the part* (B B′) *described on the other.*†

Case 1. Let A B be perpendicular to B C. Bisect A B in D, draw D E= A D or D B; produce, making D F=D E: from D describe arcs B E, A F.

Let E F, in concert with arcs B E, A F, revolve upon D in the same plane, and in direction of the arrows; and simultaneously, let A B move on B C. Let the length measured on arc B E in its movement through B be constantly equal to the length described on B C by A B.‡ Let these simultaneous movements continue until point E of arc B E meets B C, as in B′. Let B′A′ be the new position of B D A, and hence, also, of E D F. The arc B E (Euc. I. 19, Cor. b)§ would touch alone in B, B′.

(*Fig.* 1.)



*As otherwise, *intuition* would be *superfluous*, and *superfluity* is certainly not one of the attributes of the DEITY. How absurd, therefore, in certain mathematicians to assert that the *demonstrable* theorem, prop. B, of this work is self-manifest.

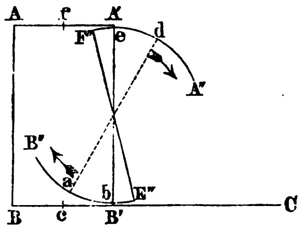
†In contemplating this demonstration, earnest attention must be paid to Def. B, cor. 2.

‡It will ease the mind if *equable* motion be conceived in each of the two simultaneous movements.

§This cor., which occurs in the author's manuscript, will not be disputed by any.

(a.) No two points in B E pass through any one in B C. For, conceive that a, b, (fig. 2) in B″ E″ pass through B′ in B C. Then, because the arc a b must pass through B′, whilst the moving line remained in position A′ B′; therefore B′ would not have moved on B C equally with arc B″ E″ through B′; which (Hyp.) is impossible. Wherefore the assumption is false. Neither can two points c, B′ in B C (fig. 2) move through any one point B′ in arc B″ E″; for then likewise, B C and arc B E (fig. 1) would not have moved equally through B.

(*Fig.* 2.)



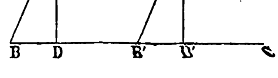
(b.) No two points in A F (fig. 1) pass through any one in A A′. Suppose that d, e, (fig. 2) pass through A′ in A A′. Then, since the arc d e must pass through A′, draw d a: and because d e passes through A′, therefore a b, which is equal to d e, passes through B′, which (Paragraph a) is impossible. Neither can any two points in A A′ (fig. 1) move through any one in A F. For, suppose that f and A′, (fig. 2) and hence line f A′ pass through A′ in arc A″ F″; therefore the corresponding portion c B′ on B B′ passes through B′ in arc B″ E″, which (Par. a) is impossible.

(c.) Now, the function performed by B E on B B′, and by A F on A A′, is that of exact coincidence, or the establishment of the equality of B E, B B′, and of A F, A A′ (fig. 1). For, the denial of the equality of B E, B B′, is simply the denial of the capability of coincidence of B E, B B′. Assuming, therefore, that a part of B E, as a b, is incapable of coincidence with B B′, it follows that a b moves through a single point in B B′, which (Par. a) is impossible. Similarly is it proved impossible for a part c d of B B′ to be incapable of coincidence with B E. Wherefore, every element of B E coincides with an equal and corresponding element of B B′, and therefore [Def. A, Schol.) B E, B B′ coincide throughout, and wherefore (Def. B) B E=B B′.

(d.) Also, arc A F=A A′ (fig. 1). For, the denial of this is simply the denial of the capability of coincidence of A F, A A′. Let e f therefore be assumed a part of A F that is incapable of coincidence with A A′; and it follows that e f passes through a single point in A A′, which (Par. b) is impossible. Similarly is it proved impossible for a part g h of A A′ to be incapable of coincidence with A F; therefore every element of A F coincides with an equal and corresponding element of A A′, and wherefore (Def. A, Schol.) A F, A A′ coincide throughout; and therefore (Def. B) A F=A A′.

(e.) Now, since arc A F=A A′, and arc B E=B B′; also, since arc A F =arc B E,* therefore A A′=B B′. Similarly is it proved that if A B move on B C for other lengths equal each to B B′, then also the lines traced by A are equal respectively to the lengths described on B C by B: and therefore the points A, B describe equal lengths.

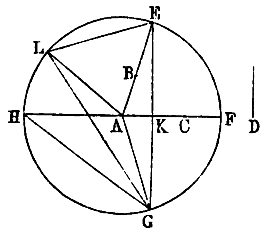
Case 2.—Let A B be oblique to B C; draw A D perpendicular to B C, and conceive triangle A B D to move upon B C in the same plane. Let A A′ be the *locus* of its vertex, and B B′, D D′ the lengths described by either extremity of the base. Then (Case 1st) A A′=D D′; but D D′=B B′; therefore A A′ =B B′.



*The proof that vertical arcs are equal is simple.

Lemma.—In either of two intersecting straight lines (A B, A C), *or continuation of either, to find a point* (E), *such that any line between it and the other line shall exceed any given line* (D).*

Let B A C be acute, and let n denote the times B A C is contained completely in 4 right angles. Take A E=$\frac{1}{2}$ (D$\times n$): from A with radius A E describe circle cutting A C in F, H; draw E K perpendicular to A F, and let E K cut circle in G, and join A G. In triangle A E G (Euc. I, 5) angle A E G=A G E: then in triangles A E K, A G K, the angles A E K, A K E =A G K, A K G, each to each; and A E=A G; therefore (Euc. I, 26) angle E A K=G A K; and wherefore angle E A G=2 E A K. Make angle G A L=E A G: if L lie between H and E, join G L, L E, H G: and since E G, G L envelope E A, A L, therefore (Euc. I, 21) E G+G L>E A+A L, or 2 A E. But E G or G L=2 E K: also angle E A G+G A L are not greater than B A C$\times n$; and therefore E G+G L are not greater than E K$\times n$; and wherefore E K$\times n$>2 A E or D$\times n$: therefore E K>D. If angle B A C diminish so that L falls between H and G, then Euc. I, 24, is quoted instead of Euc. I, 26. The solution of the case wherein B A C is obtuse is involved in the foregoing. The solution of the case wherein B A C is a right angle is effected simply by taking A E greater than D.

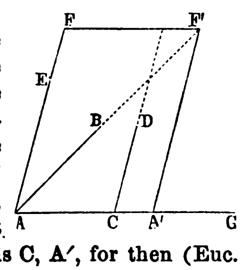


PROPOSITION B THEOREM.

[Being the celebrated so-called 12th Axiom of Euclid.]

If a straight line (A C) *meet two other straight lines* (A B, C D) *which are in the same plane, so as to make the two interior angles* (B A C, A C D) *on the same side of it, taken together, less than two right angles, these two straight lines* (A B, C D) *shall at length meet upon that side, if sufficiently produced.*

Draw A E making angle E A C+A C D=2 right angles. In A E or its production find (*Lemma*) F, such that any line between F and A B shall exceed A C. Produce A C to G. Let A F move on A C as enunciated in prop. A, and until F falls on A B or its production, as F′; where A′ F′ is the new position of A F, and F F′ the *locus* of F. Because (const.) F F′>A C; and because (prop. A) F F′=A A′; therefore A A′>A C. Since angle E A C+A C D=2 right angles; and since angle A C D+D C A′=2 right angles, therefore angle D C G=E A C=F′ A′ G; and wherefore C D, A F′ are *parallel;* for, if C D, A′ F′ meet towards D, F′, then (Euc. I, 16) angle F′ A′ G>D C A′; but this is not so. Neither can C D, A′ F′ meet towards C, A′, for then (Euc. I, 16) angle vertical to D C A′ would exceed angle vertical to F′ A′ G; but it does not. Therefore C D, A′ F′ are *parallel*: and since A′ F′ meets A B



*This Lemma was assumed by Proclus; also by Aristotle in his attempt "to establish that the world is finite."

produced, C D shall likewise meet A B, or production of A B within the figure F A A′ F′: which was to be demonstrated. Therefore, "*If a straight line meet,*" &c.

APPENDIX.—NOTE A.

On the beautiful philosophical relation between the mode of proof in prop. A *and the establishment of the knowledge of Parallelism.*

(a.) *Coincidence*, or the *capability* of *coincidence*, and known as *equality*, is the basis of all geometric knowledge. *Equality* is, in all cases save one, manifested in the *direct* manner; that is, in either of the first four forms set forth in cor. to Def. A. Thus, by *direct* coincidence is established the equality or inequality of the sides, angles or areas of the two magnitudes in Euc. I, 4, 5, 6, 8, 18, 19, 24, 25, 26, 35, &c., to 47: also Euc. III, 20, 21, 24, 26, &c.; as well as the knowledge of all propositions which ultimately rest on Euc. I, 4, as basis. In these all, also, the *idea* is composed of two *sub-ideas* or *terms*, which are unrelated except by comparison, or by possessing the property of equality. Alone *Parallelism* presents a single conception—consists of but one *term*, that of *non-concurrence.* Other subjects present to the mind two homogeneous magnitudes and their relations; and when the subject is the equality or inequality of two lines or angles, these are related with surfaces or solids. In other subjects the *datum* is *equality* or *inequality*, or their combination; but in *parallelism* the *datum* simply is that *the lines shall never meet.*

(b.) To discover *new immutable* truths, the human mind requires at least two truths to work with (*vide Elucid. to Ax.*): and since *parallelism* commands but a single conception—that of *non-concurrence*—therefore in the establishment of the laws of parallels, the 4 *direct* forms of coincidence yield to the 5th or *indirect* form (see Def. A); and hence, *two identical auxiliaries* must be used, and be simultaneously superposed on either parallel. Such *auxiliaries* are alone found in vertical arcs of the circumference. Thus, the clasping of either parallel by the equal arcs A F, B E, and the simultaneous coincidence of these arcs with A F′, B B′, fulfill the functions of the essential principle of *direct* coincidence.

(c.) The identity of the circumference in all its parts compels the eternal continuance of any law established by the least rotation of the circumference. Thus is the circumference the unique symbol of that *infinity* which is embraced in *Parallelism.** By no properties of straight lines alone could the knowledge of parallels be established, for the straight line involves only *finite* considerations, and the *direct* forms of coincidence. Whilst, also, on the one hand, the circumference in its form contains a certain *infinite law*, and in its being a *finite whole*, (and therefore every arc a portion of a *finite whole*) lies within the comprehension of the human mind; on the other hand, every straight line is but a portion of an *infinite whole*, and hence, of itself, fails to present to the mind those certain *infinite laws* which have a peculiar existence in *parallelism.*† Thus, by the consideration of the equality of *finites*, viz.: B E,

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*The demonstrative knowledge of many properties of the circle is still veiled from man. Some of these occur in the revolution of a radius about the centre.

†We can therefore account for the failures of those forty and more ancient and modern attempts wherein was alone employed the straight line. T. Perronet Thompson, alone of all Geometers, appealed to the circle, but in vain, because not having employed the circumference in that *auxiliary* method given in prop. A. See T. P. Thompson's Pamphlet, 1856.

B B′, (fig. prop. A) do we establish a knowledge of an *Infinite*, viz.: *Parallelism.* Thus, also, universally, from the comparison of the equality of *finites* may be evolved some positive knowledge of the corresponding homeogeneous *Infinites*, whether in *Deity*, *space*, *time* or *degree.*

(d.) A remarkable sophism or parallogism committed by a certain member of the *American Institute* of New York City is worthy of record. This gentleman denied the capability of coincidence of a curve and straight line (see B E, B B′, fig. prop. A) either in whole or part, but admitted their equality. This sophism or parallogism is refuted by Ax. I and II. Thus (Ax. I) B E, B B′, are either equal or unequal, but (Ax. II) not both. If the equality of B E, B B′ be admitted, (Def. B) the capability of coincidence of B E, B B′ is thereby admitted ; whilst if the inequality of B E, B B′ be admitted, from the very signification of inequality (Def. B, cor. 2, par. a) B E, B B′ if superposed, would coincide in part. Thus, the vague *principles* of Metaphysics fall before the two *axioms* of *Geometry.*

NOTE B.

On the establishment of a positive knowledge of the Unconditioned or Infinite in Space. Being a partial refutation of Sir Wm. Hamilton's philosophy of the Unconditioned or Infinite.

(a.) The knowledge established in prop. B is *positive* because *demonstrative;* it is *entire*, and is that of an *Infinite Whole*, for *parallelism* involves INFINITE SPACE both in the direction of the lines and their distance apart. Such is admitted even by the Hamiltonians. Saith a disciple of Hamilton, "to deduce the properties of two lines postulated as parallels involves a *direct* dealing with the *positive idea of infinity*—a task utterly beyond the reach of our faculties."* Sir William, himself, conceives that "The unconditioned is incognizable and inconceivable ;" that "the mind can conceive, and consequently, can know, only the *limited and the conditionally limited*," &c., and that "The result is the same, whether we apply the process to limitation in *space*, in *time*, or in *degree.*"†

(b.) Hamilton, whilst refuting the German and French metaphysicians who impiously ascribed a *complete* knowledge of the *Infinite or Absolute* to man, erred in denying to man *any* knowledge of the *Infinite.* Between the false Hamiltonian philosophy and Absolutism lies that true philosophy which yields to man *some positive* knowledge of the *Infinite or Absolute.* N. B.—In *parallelism* alone has man acquired a *complete positive* knowledge of an *Infinite.*

Schol.—The laws of geometrical equality being eternal—not the result of POWER, such as the physical laws of the universe—therefore the WHY of every geometrical law seems possible to man. Besides, "Is not the conception of space common to all men, and does it not contain in it all that is necessary for a science of Geometry ? If so, there should be nothing disputable or open to cavil in the minds of competent thinkers."‡

The student should distinguish between *theoretical* and *actual Infinites:* thus, *asymptotes* and *numbers* are infinite only in theory, &c., &c.

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*Lond. Edin. and Dub. Phil. Mag., 1857, Vol. XIII. page 413, as quoted from "Nichol's Cyclopædia of the Physical Sciences."

†Philosophy of the Unconditioned. (See Hamilton's *Discussions.*)

‡Dr. Day on *Parallels.* Lond. Edin. and Dub. Phil. Mag., 1857, Vol. XIII, page 156.